Circle your Instructor:
Faudree, Williams, Zirbes

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Math 251 Fall 2017 **Quiz #5, October 11th**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (6 pts.) Differentiate the following functions.

(a) $f(t) = 5^{2t^2} = (5)^{(2t^2)}$

$f'(t) = (\ln 5)\, 5^{2t^2} \cdot \frac{d}{dt}\left[2t^2\right]$

$= (\ln 5)\, 5^{2t^2} \cdot 4t$

$\boxed{= (4\ln 5) t \cdot 5^{2t^2}}$

(b) $f(\theta) = \theta \sin\theta \cos\theta$

$f'(\theta) = 1 \cdot (\sin\theta\cos\theta) + \theta \cdot \underbrace{\frac{d}{d\theta}(\sin\theta\cos\theta)}$

$= \sin\theta\cos\theta + \theta \cdot \left[\sin\theta \cdot (-\sin\theta) + \cos\theta \cdot \cos\theta\right]$

$\boxed{= \sin\theta\cos\theta + \theta\left(\cos^2\theta - \sin^2\theta\right)}$

Exercise 2. (6 pts.) find the derivatives of the following functions.

(a) $g(x) = \sec^3(5x) = \left[\sec(5x)\right]^3$

$g'(x) = 3\left[\sec(5x)\right]^2 \underbrace{\left(\sec(5x)\tan(5x)\right) \cdot 5}_{\frac{d}{dx}(\sec(5x))}$

$= \boxed{15\sec^3(5x)\tan(5x).}$

(b) $f(x) = e^{x \csc x}$

$f'(x) = e^{x\csc x} \cdot \frac{d}{dx}(x\csc x)$

$= e^{x\csc x} \cdot \left[1 \cdot \csc x + x \cdot (-\cot x \csc x)\right]$

$\boxed{= \csc x(1 - x\cot x)\, e^{x\csc x}}$

Exercise 3. (4 pts.) For what values of x does $y = \sqrt{x^2 + x}$ have a horizontal tangent?

$y = (x^2 + x)^{1/2}$

$y' = \frac{1}{2}(x^2+x)^{-1/2}(2x+1)$

$= \frac{2x+1}{2\sqrt{x^2+x}} = 0$

we want x where $y' = 0$ (or slope is horizontal.)

Thus, we require:

$2x + 1 = 0$ or $x = -1/2$.

But, $x = -1/2$ is not in the domain of the derivative y'.

ans: y has no horizontal tangents

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point

Exercise 4. (4 pts.) Find an equation of the tangent line to the curve $y = \frac{10}{(\tan x + 2)^2}$ at the point (0,**5**).

$$y = 10(\tan x + 2)^{-2}$$

$$y' = -20(\tan x + 2)^{-3}(\sec^2 x)$$

$$= \frac{-20\sec^2 x}{(\tan x + 2)^3}$$

$$y'(0) = \frac{-20\sec^2 0}{(\tan 0 + 2)^3} = \frac{-20}{8} = \frac{-5}{2} = m \leftarrow \text{slope}$$

tangent line:

$$y - 5 = \frac{-5}{2}(x - 0)$$

$$\boxed{y = \frac{-5}{2}x + 5}$$

Correction: The point should have been $(0, 5/2)$. So the line should have been

$$y = \frac{-5}{2}x + \frac{5}{2}.$$

Exercise 5. (5 pts.) Find the 50th derivative of $y = \cos(4x)$.

(a) Find the first four derivatives of $y = \cos(4x)$.

$$y' = -4\sin(4x)$$

$$y'' = -4^2\cos(4x)$$

$$y''' = 4^3\sin(4x)$$

$$y^{(4)} = 4^4\cos(4x)$$

(b) Using your answer to (a), find the 50th derivative of $y = \cos(4x)$.

$$\begin{array}{r} 12 \\ 4\overline{)50} \\ \underline{4} \\ 10 \\ \underline{8} \\ 2 \end{array}$$

$\leftarrow$ 2 derivatives past $4 \cdot 12 = 48$.

Every 4 derivatives, we get back to $\cos(4x)$. So

$$y^{(50)} = -4^{50}\cos(4x).$$